Math 142 - Pre-Calculus Review Problems

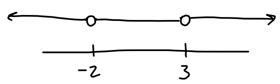
courtesy: Kendra Kilmer

(from Fall 2017 Math 140 WIR)

1. State the following in interval notation.

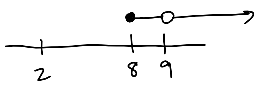
(a) $x \neq 3$ and $x \neq -2$

$\boxed{(-\infty, -2) \cup (-2, 3) \cup (3, \infty)}$



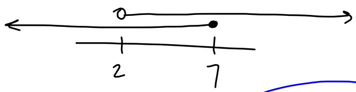
(b) $x \geq 8$ and $x \neq 2$ and $x \neq 9$

$\boxed{[8, 9) \cup (9, \infty)}$



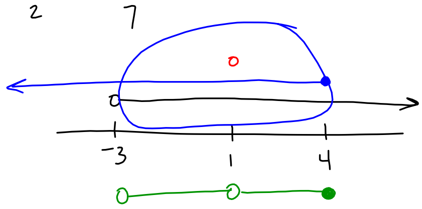
(c) $x \leq 7$ or $x > 2$

$\boxed{(-\infty, \infty)}$

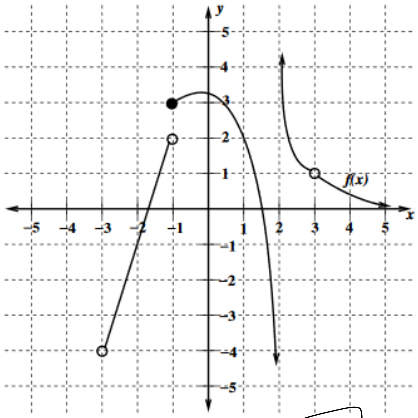


(d) $x > -3$ and $x \leq 4$ and $x \neq 1$

$\boxed{(-3, 1) \cup (1, 4]}$



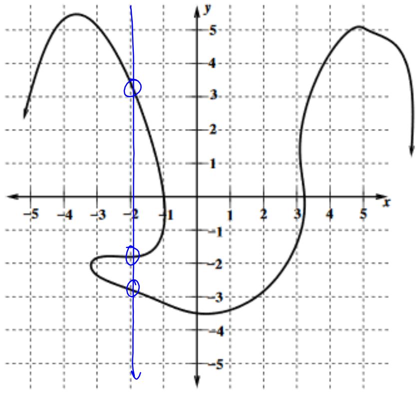
2. Does each of the following graphs represent a function? If so, find the domain and range.



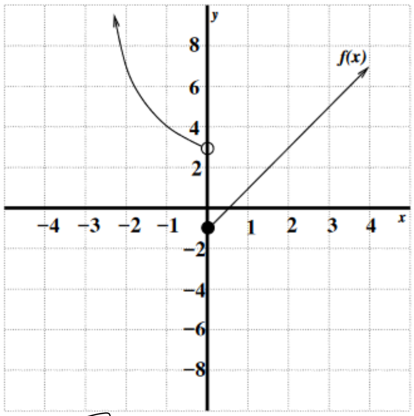
(a) Yes

Domain: $(-3, 2) \cup (2, 3) \cup (3, \infty)$

Range: $(-\infty, \infty)$



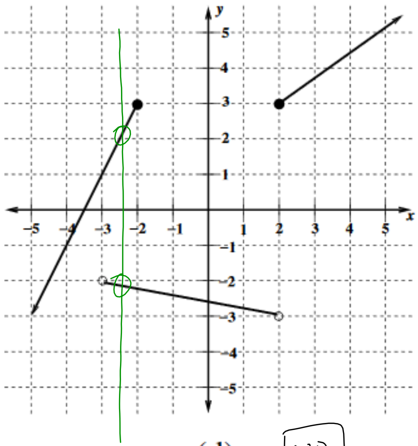
(b) NO



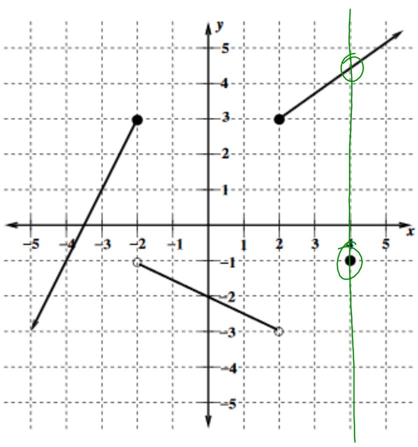
(c) Yes

Domain: $(-\infty, \infty)$

Range: $[-1, \infty)$

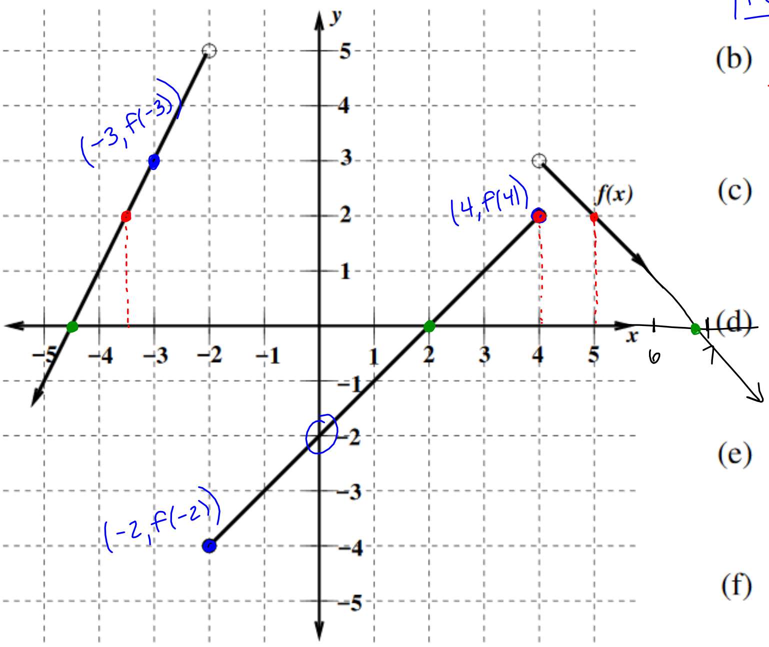


(d) NO



(e) NO

3. Use the graph below to find the following:



(a) $f(-3)$, $f(-2)$, $f(4)$

$\boxed{f(-3)=3}$ $\boxed{f(-2)=-4}$ $\boxed{f(4)=2}$

(b) the value(s) of x such that $f(x)=2$

$\boxed{x\approx -3.5,\ x=4,\ x=5}$

(c) the zeros of $f(x)$

$\boxed{x\approx -4.5,\ x=2,\ x\approx 6.8}$

(d) the y-intercept of $f(x)$

$\boxed{-2}$

(e) domain of $f(x)$

$\boxed{(-\infty,\infty)}$

(f) range of $f(x)$

$\boxed{(-\infty,5)}$

4. If $f(x) = 3x - 10$ and $g(x) = 2x^2 - 7x + 3$, find the following:

(a) $f(2)$ $= 3(2) - 10$

$= \boxed{-4}$

(b) $g(-3)$ $= 2(-3)^2 - 7(-3) + 3$

$= 18 + 21 + 3$

$= \boxed{42}$

(c) $f(a-7)$ $= 3(a-7) - 10$

$= 3a - 21 - 10$

$= \boxed{3a - 31}$

(d) $g(2a+4)$ $= 2(2a+4)^2 - 7(2a+4) + 3$

$= 2(4a^2 + 16a + 16) - 14a - 28 + 3$

$= 8a^2 + 32a + 32 - 14a - 25$

$= \boxed{8a^2 + 18a + 7}$

$(2a+4)^2$

$= (2a+4)(2a+4)$

$= 4a^2 + 8a + 8a + 16$

$= 4a^2 + 16a + 16$

(e) $g(a+h) - g(a)$

$= \left(2(a+h)^2 - 7(a+h) + 3\right) - \left(2a^2 - 7a + 3\right)$

$= \left(2(a^2 + 2ah + h^2) - 7a - 7h + 3\right) - 2a^2 + 7a - 3$

$= \cancel{2a^2} + 4ah + 2h^2 \cancel{-7a} - 7h \cancel{+3} \cancel{-2a^2} \cancel{+7a} \cancel{-3}$

$= \boxed{4ah + 2h^2 - 7h}$

$(a+h)^2$

$= (a+h)(a+h)$

$= a^2 + ah + ah + h^2$

$= a^2 + 2ah + h^2$

5. For each of the following quadratic functions, find the (i) vertex form of the function, (ii) the zeros of the function, and (iii) the domain and range of the function.

$\boxed{f(x) = a(x-h)^2 + k}$

(a) $g(x) = -6x^2 - 39x + 21$

(i) $a = -6$, $b = -39$, $c = 21$

$h = \frac{-b}{2a} = \frac{+39}{2(-6)} = \frac{-13}{4}$

$k = g(-\frac{13}{4}) = -6(-\frac{13}{4})^2 - 39(-\frac{13}{4}) + 21 = \frac{675}{8}$

$\boxed{g(x) = -6(x + \frac{13}{4})^2 + \frac{675}{8}}$

(ii) $-6x^2 - 39x + 21 = 0$

$-3(2x^2 + 13x - 7) = 0$

$-3(2x-1)(x+7) = 0$

$2x - 1 = 0$, $2x = 1$, $\boxed{x = 1/2}$

$x + 7 = 0$, $\boxed{x = -7}$

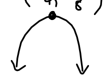
Check: $(2x-1)(x+7)$

$= 2x^2 + 14x - x - 7$

$= 2x^2 + 13x - 7$

(iii) Domain: $(-\infty, \infty)$

Range: $(-\infty, \frac{675}{8}]$



(b) $m(x) = 2x^2 - 7x + 6$

(i) $a = 2$, $b = -7$, $c = 6$

$h = \frac{-b}{2a} = \frac{-(-7)}{2(2)} = \frac{7}{4}$

$k = m(7/4) = 2(7/4)^2 - 7(7/4) + 6 = -\frac{1}{8}$

$\boxed{m(x) = 2(x - \frac{7}{4})^2 - \frac{1}{8}}$

(ii) $2x^2 - 7x + 6 = 0$

$(2x-3)(x-2) = 0$

$2x - 3 = 0$, $2x = 3$, $\boxed{x = 3/2}$

$x - 2 = 0$, $\boxed{x = 2}$

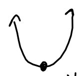
Check: $(2x-3)(x-2)$

$= 2x^2 - 4x - 3x + 6$

$= 2x^2 - 7x + 6$ ✓

(iii) Domain: $(-\infty, \infty)$

Range: $[-1/8, \infty)$



(c) $f(x) = 7x^2 - 4x + 9$

(i) $a = 7$, $b = -4$, $c = 9$

$h = \frac{-b}{2a} = \frac{-(-4)}{2(7)} = \frac{4}{14} = \frac{2}{7}$

$k = f(2/7) = 7(2/7)^2 - 4(2/7) + 9 = \frac{59}{7}$

$\boxed{f(x) = 7(x - \frac{2}{7})^2 + \frac{59}{7}}$

(ii) $7x^2 - 4x + 9 = 0$

$\cancel{(7x-3)(x-3)}$

we cannot factor into linear terms ⇒ Quadratic Formula

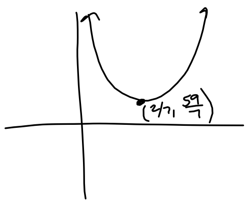
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(7)(9)}}{2(7)}$

$= \frac{4 \pm \sqrt{-236}}{14}$ ⇒ $\boxed{\text{No real roots / zeros}}$

(iii) Domain: $(-\infty, \infty)$

Range: $[59/7, \infty)$



6. For a particular lamp, the consumers will not purchase it at a price of \$230 but for every \$8 decrease in price the consumers are willing to purchase 24 more lamps.

(a) Find the demand equation.

(x, p) quantity, unit price

$(0, 230)$ $(24, 222)$ b

$m = \frac{\Delta p}{\Delta x} = \frac{222-230}{24-0} = \frac{-8}{24} = -\frac{1}{3}$

$y = mx + b$

$p = mx + b$

$\boxed{p = -\frac{1}{3}x + 230}$

(b) If the demand function determines the selling price, what is the revenue function?

$R(x) = (\text{selling price})\,x$

$R(x) = (-\frac{1}{3}x + 230)x$

$\boxed{R(x) = -\frac{1}{3}x^2 + 230x}$

(h, k)

(c) What is the selling price when the revenue is maximized?

$a = -1/3$

$b = 230$

$h = \frac{-b}{2a} = \frac{-230}{2(-1/3)} = 345$

$k = R(345) = -\frac{1}{3}(345)^2 + 230(345) = 39675$

A maximum revenue of \$39,675 occurs when they sell 345 lamps.

$p = -\frac{1}{3}(345) + 230$

$= \boxed{\$115}$

(d) If each lamp costs \$20 to make and the company has a fixed cost of \$1000, what is the cost function?

$C(x) = cx + F$ $\quad c = 20$

$F = 1000$

$\boxed{C(x) = 20x + 1000}$

(e) How many lamps should they make and sell to maximize their profit?

$P(x) = R(x) - C(x)$

$= -\frac{1}{3}x^2 + 230x - (20x + 1000)$

$= -\frac{1}{3}x^2 + 230x - 20x - 1000 = -\frac{1}{3}x^2 + 210x - 1000$

(h, k)

$a = -\frac{1}{3}$

$b = 210$

$c = -1000$

$h = \frac{-b}{2a} = \frac{-210}{2(-1/3)}$

$= \boxed{315 \text{ lamps}}$

$k = P(315)$

$= -\frac{1}{3}(315)^2 + 210(315) - 1000$

$= \$32,075$

(f) How many lamps should they make and sell to break even?

$R(x) = C(x)$

$R(x) - C(x) = 0$

$P(x) = 0$

$-\frac{1}{3}x^2 + 210x - 1000 = 0$

$x = \frac{-210 \pm \sqrt{210^2 - 4(-1/3)(-1000)}}{2(-1/3)}$

$x = \frac{-210 + \sqrt{128300/3}}{-2/3}$ $\quad x = \frac{-210 - \sqrt{\frac{128300}{3}}}{-2/3}$

$\approx \boxed{5 \text{ lamps}}$ $\quad \approx \boxed{625 \text{ lamps}}$

7. Find the domain of each of the following functions:

(a) $f(x) = \dfrac{5x-7}{8x-3}$ rational

$8x-3 \neq 0$

$8x \neq 3$

$x \neq 3/8$

$3/8$

$\boxed{(-\infty, 3/8) \cup (3/8, \infty)}$

(b) $h(x) = \dfrac{8x-17}{3x^2-8x+1}$

$\frac{0}{10} = 0$ $\quad \frac{10}{0}$ not defined

$3x^2-8x+1 \neq 0$

$\cancel{(3x-1)(x-1)}$

$= \cancel{3x^2-4x+1}$

$ax^2+bx+c \neq 0$ $\quad a=3$, $b=-8$, $c=1$

$x \neq \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\neq \dfrac{-(-8) \pm \sqrt{(-8)^2-4(3)(1)}}{2(3)}$

$\neq \dfrac{8 \pm \sqrt{52}}{6}$

$x \neq \dfrac{8-\sqrt{52}}{6}, \quad x \neq \dfrac{8+\sqrt{52}}{6}$

$\dfrac{8-\sqrt{52}}{6}$ $\quad \dfrac{8+\sqrt{52}}{6}$

$\boxed{\left(-\infty, \frac{8-\sqrt{52}}{6}\right) \cup \left(\frac{8-\sqrt{52}}{6}, \frac{8+\sqrt{52}}{6}\right) \cup \left(\frac{8+\sqrt{52}}{6}, \infty\right)}$

(c) $k(x) = \dfrac{x-2}{x^4+x^3-6x^2}$

$x^4+x^3-6x^2 \neq 0$

$x^2(x^2+x-6) \neq 0$

$x^2(x+3)(x-2) \neq 0$

$x^2 \neq 0$ $\quad x+3 \neq 0$ $\quad x-2 \neq 0$

$x \neq 0$ $\quad x \neq -3$ $\quad x \neq 2$

$-3 \quad 0 \quad 2$

$\boxed{(-\infty, -3) \cup (-3, 0) \cup (0, 2) \cup (2, \infty)}$

← even root

(d) $m(x) = \dfrac{\sqrt[4]{3x-5}}{\sqrt[7]{5x-9}}$ odd root

$3x-5 \geq 0$

$3x \geq 5$

$x \geq 5/3$

$5x-9 \neq 0$

$5x \neq 9$

$x \neq 9/5$

$5/3 \quad 9/5$

$\boxed{[5/3, 9/5) \cup (9/5, \infty)}$

$\sqrt{4} = 2$

$\sqrt{-4} = \cancel{\ }$

$\sqrt[3]{8} = 2$

$\sqrt[3]{-8} = -2$

(e) $T(x) = \begin{cases} \dfrac{5x-7}{9x^2-9} & \text{if } x < 0 \\ \sqrt[8]{5x+9} & \text{if } 0 \leq x \leq 2 \\ \sqrt[3]{3x-20} & \text{if } x > 4 \end{cases}$

Just looking @ what values we have a formula for first.

$-1 \quad 0 \quad 2 \quad 4$

1st branch

$\dfrac{5x-7}{9x^2-9}$

$9x^2-9 \neq 0$

$9(x^2-1) \neq 0$

$9(x+1)(x-1) \neq 0$

$x \neq -1$, $\cancel{x \neq 1}$

this is an issue for $T(x)$ since $x=-1$ is on the int. $x<0$

this is not an issue for $T(x)$ since $x=1$ is not on the interval $x<0$

2nd branch

even root $\sqrt[8]{5x+9}$

$5x+9 \geq 0$

$5x \geq -9$

$x \geq -9/5$

This doesn't come into play here since $0 \leq x \leq 2$ is to the right of $x = -\frac{9}{5}$.

3rd branch

odd root $\sqrt[3]{3x-20}$

defined for all values of x

$\boxed{(-\infty, -1) \cup (-1, 2] \cup (4, \infty)}$

$\cancel{[-1, 0] \cup (0, 2]}$

this is equivalent to $(-1, 2]$ but it is easier to write it as $(-1, 2]$

(f) $f(x) = e^{\frac{x}{x-4}}$

$x-4 \neq 0$

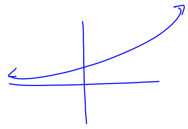
$x \neq 4$

$\boxed{(-\infty, 4) \cup (4, \infty)}$

(g) $g(x) = \dfrac{x-2}{2^{5x-4}}$

$2^{5x-4} \neq 0$

no value of x for which 2^{5x-4} touches the x-axis.



$\boxed{(-\infty, \infty)}$

8. Perform the indicated operations and simplify

(a) $\dfrac{x+2}{x-4} \cdot \dfrac{x+7}{2x-9} = \boxed{\dfrac{(x+2)(x+7)}{(x-4)(2x-9)}}$

(b) $\dfrac{\frac{x+2}{x+7}}{\frac{2x-7}{3x+2}} = \dfrac{x+2}{x+7} \cdot \dfrac{3x+2}{2x-7}$

$= \boxed{\dfrac{(x+2)(3x+2)}{(x+7)(2x-7)}}$

(c) $\dfrac{x^2}{7x-8} + \dfrac{x-2}{x+5}$

$\dfrac{(x+5)}{(x+5)} \cdot \dfrac{x^2}{(7x-8)} + \dfrac{(x-2)}{(x+5)} \dfrac{(7x-8)}{(7x-8)}$

$\dfrac{x^2(x+5)+(x-2)(7x-8)}{(x+5)(7x-8)}$

$= \dfrac{x^3+\underline{5x^2}+\underline{7x^2}-22x+16}{(x+5)(7x-8)}$

$= \boxed{\dfrac{x^3+12x^2-22x+16}{(x+5)(7x-8)}}$

$\dfrac{2}{7} \cdot \dfrac{3}{5} = \dfrac{2 \cdot 3}{7 \cdot 5}$

$\dfrac{\frac{2}{7}}{\frac{3}{5}} = \dfrac{2}{7} \cdot \dfrac{5}{3} = \dfrac{2 \cdot 5}{7 \cdot 3}$

$\dfrac{5}{5} \cdot \dfrac{2}{7} + \dfrac{3}{5} \cdot \dfrac{7}{7}$

$\dfrac{5 \cdot 2 + 3 \cdot 7}{5 \cdot 7}$

9. For each of the following functions, evaluate and simplify $\dfrac{f(x+h)-f(x)}{h}$

(a) $f(x) = 3x^2 - 9x + 10$

$f(x+h) = 3(x+h)^2 - 9(x+h) + 10$
$= 3(x+h)(x+h) - 9x - 9h + 10$
$= 3(x^2 + 2hx + h^2) - 9x - 9h + 10$
$= 3x^2 + 6hx + 3h^2 - 9x - 9h + 10$

$\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{\cancel{3x^2} + 6hx + 3h^2 \cancel{-9x} - 9h \cancel{+10} \cancel{-3x^2} \cancel{+9x} \cancel{-10}}{h}$

$= \dfrac{6hx + 3h^2 - 9h}{h} = \dfrac{\cancel{h}(6x + 3h - 9)}{\cancel{h}} = \boxed{6x + 3h - 9,\ h \neq 0}$

(b) $f(x) = \dfrac{7}{3x-5}$

$f(x+h) - f(x)$
$= \dfrac{7}{3(x+h)-5} - \dfrac{7}{3x-5}$
$= \dfrac{(3x-5)}{(3x-5)} \dfrac{7}{3x+3h-5} - \dfrac{7}{3x-5} \dfrac{(3x+3h-5)}{(3x+3h-5)}$
$= \dfrac{7(3x-5) - 7(3x+3h-5)}{(3x-5)(3x+3h-5)}$
$= \dfrac{\cancel{21x} \cancel{-35} \cancel{-21x} - 21h \cancel{+35}}{(3x-5)(3x+3h-5)}$
$= \dfrac{-21h}{(3x-5)(3x+3h-5)}$

$\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{\dfrac{-21h}{(3x-5)(3x+3h-5)}}{h}$

$= \dfrac{-21h}{(3x-5)(3x+3h-5)} \cdot \dfrac{1}{h}$

$= \dfrac{-21\cancel{h}}{\cancel{h}(3x-5)(3x+3h-5)} = \boxed{\dfrac{-21}{(3x-5)(3x+3h-5)},\ h \neq 0}$

(c) $f(x) = \sqrt{3x-8}$

$f(x+h) - f(x)$
$= \sqrt{3(x+h)-8} - \sqrt{3x-8}$
$= \sqrt{3x+3h-8} - \sqrt{3x-8}$

$\dfrac{f(x+h)-f(x)}{h}$

$= \dfrac{(\sqrt{3x+3h-8} - \sqrt{3x-8})}{h} \dfrac{(\sqrt{3x+3h-8} + \sqrt{3x-8})}{(\sqrt{3x+3h-8} + \sqrt{3x-8})}$

$(\sqrt{3x+3h-8} - \sqrt{3x-8})(\sqrt{3x+3h-8} + \sqrt{3x-8})$
$= \sqrt{3x+3h-8}\sqrt{3x+3h-8} + \cancel{\sqrt{3x-8}\sqrt{3x+3h-8}} - \cancel{\sqrt{3x-8}\sqrt{3x+3h-8}} - \sqrt{3x-8}\sqrt{3x-8}$
$= 3x + 3h - 8 - (3x-8)$
$= \cancel{3x} + 3h \cancel{-8} \cancel{-3x} \cancel{+8}$
$= 3h$

$= \dfrac{3\cancel{h}}{\cancel{h}(\sqrt{3x+3h-8} + \sqrt{3x-8})}$

$= \boxed{\dfrac{3}{\sqrt{3x+3h-8} + \sqrt{3x-8}},\ h \neq 0}$

$\sqrt{x}\sqrt{x}$
$= x^{1/2} x^{1/2}$
$= x^{1/2 + 1/2}$
$= x^1 = x$

10. If $f(x) = \begin{cases} -3x^2+4x-7 & \text{if } x<2 \\ 3^x & \text{if } 2 \le x < 5 \\ 7x-9 & \text{if } x>7 \end{cases}$, find the following.

(a) $f(0)$ $= -3(0)^2+4(0)-7$
$= \boxed{-7}$

(b) $f(2)$ $= 3^2 = \boxed{9}$

(c) $f(5)$ $\boxed{\text{does not exist}}$ 5 is not on any of the three intervals
$x<2$ $\quad 2 \le x < 5$ $\quad x>7$

(d) $f(8)$ $= 7(8)-9 = \boxed{47}$

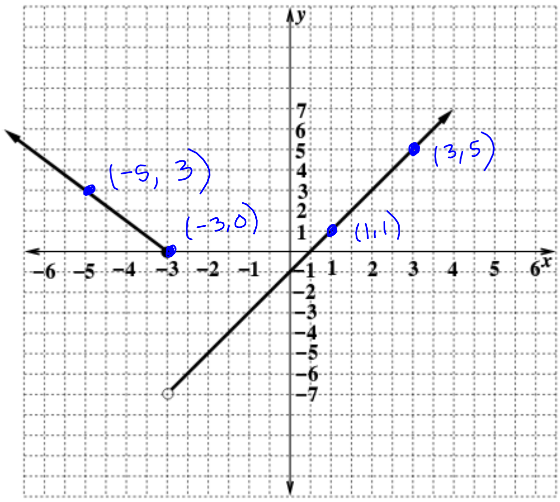
11. Write $f(x) = |2x - 10|$ as a piecewise defined function.

$$\boxed{|2x-10| = \begin{cases} -(2x-10), & x<5 \\ 2x-10, & x \geq 5 \end{cases}}$$

$$|x| = \begin{cases} -x, & x<0 \\ x, & x \geq 0 \end{cases}$$

$$2x - 10 \geq 0$$
$$2x \geq 10$$
$$x \geq 5$$

12. Find a formula for the function f graphed below.



$$f(x) = \begin{cases} -\frac{3}{2}x - \frac{9}{2} & , \ x \le -3 \\ 2x-1 & , \ x > -3 \end{cases}$$

1st branch

(-5,3) (-3,0)

$m = \frac{0-3}{-3+5} = \frac{-3}{2} = -\frac{3}{2}$ m

$y = mx+b$

$3 = -\frac{3}{2}(-5)+b$

$3 = \frac{15}{2}+b$

$3-\frac{15}{2} = b$

$-\frac{9}{2} = b$

$y = -\frac{3}{2}x - \frac{9}{2}$

2nd branch

(1,1) (3,5)

$m = \frac{5-1}{3-1} = \frac{4}{2} = 2$ m

$y = mx+b$

$1 = 2(1)+b$

$\cancel{1 = 2+b}$

$-1 = b$

$y = 2x-1$

13. Sketch a graph of $f(x) = \begin{cases} -\frac{1}{2}x^2+7 & \text{if } -4 \le x < 0 \\ -\frac{5}{2}x+6 & \text{if } 2 < x \le 4 \\ \frac{1}{4}x+5 & \text{if } x > 4 \end{cases}$

1st branch

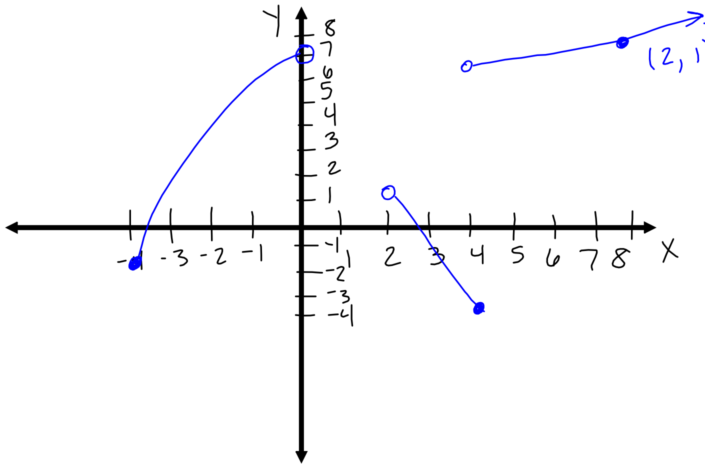
(-4, -1) (0, 7)

2nd branch

(2, 1) (4, -4)

3rd branch

(4, 6) (8, 7)



14. A daycare center charges \$20 to watch a child for a four hour period. For each minute during the 5th hour, they charge \$0.50 per minute. For each minute in excess of 5 hours,they charge \$1 per minute. If $C(x)$ is the total daily cost of putting your child in the daycare for x minutes, ~~which of the following~~ what is $C(x)$?

$$\boxed{C(x) = \begin{cases} 20 & , 0 \le x \le 240 \\ 20 + .5(x-240) & , 240 < x \le 300 \\ 20 + .5(60) + 1(x-300) & , x > 300 \end{cases}}$$

$C(241) = 20 + .5(1)$ 241-240

$C(260) = 20 + .5(20)$ 260-240

15. Simplify the following and express the answer without using radicals or negative exponents.

(a) $\dfrac{(3x)^{-2}\sqrt{x^3y}}{5(xy)^3x^{-1/2}} = \dfrac{3^{-2}x^{-2}(x^3y)^{1/2}}{5x^3y^3x^{-1/2}} = \dfrac{3^{-2}x^{-2}(x^3)^{1/2}y^{1/2}}{5x^3y^3x^{-1/2}}$

$= \dfrac{3^{-2}x^{-2}x^{3/2}y^{1/2}}{5x^3y^3x^{-1/2}}$

$= \dfrac{3^{-2}}{5}x^{-2+3/2-3+1/2}y^{1/2-3}$

$= \dfrac{1}{3^2\cdot 5}x^{-3}y^{-5/2}$

$= \boxed{\dfrac{1}{45x^3y^{5/2}}}$

(b) $\dfrac{(xyz)^3}{(\sqrt[3]{xy})^{-2}} = \dfrac{x^3y^3z^3}{\left((xy)^{1/3}\right)^{-2}}$

$= \dfrac{x^3y^3z^3}{(xy)^{-2/3}} = \dfrac{x^3y^3z^3}{x^{-2/3}y^{-2/3}}$

$= x^{3+2/3}y^{3+2/3}z^3$

$= \boxed{x^{11/3}y^{11/3}z^3}$

(c) $\dfrac{(3x^3y^6z^{-2})^{-3}}{(2xy)^2xyz^{-4}}$

$= \dfrac{3^{-3}(x^3)^{-3}(y^6)^{-3}(z^{-2})^{-3}}{2^2x^2y^2xyz^{-4}} = \dfrac{x^{-9}y^{-18}z^6}{27\cdot 4\,x^3y^3z^{-4}}$

$= \dfrac{1}{108}x^{-9-3}y^{-18-3}z^{6+4}$

$= \dfrac{1}{108}x^{-12}y^{-21}z^{10}$

$= \boxed{\dfrac{z^{10}}{108x^{12}y^{21}}}$

16. Solve the following for the exact value of x.

(a) $16^{2x+5} = 4^{3x}$

$(4^2)^{2x+5} = 4^{3x}$

$4^{4x+10} = 4^{3x}$

$4x+10 = 3x$

$\boxed{x=-10}$

(b) $\dfrac{1}{9^{3x}} = 27^{5x-7}$

$9^{-3x} = 27^{5x-7}$

$(3^2)^{-3x} = (3^3)^{5x-7}$

$3^{-6x} = 3^{15x-21}$

$-6x = 15x-21$

$\dfrac{-21x}{-21} = \dfrac{-21}{-21}$

$\boxed{x=1}$

(c) $2^x \cdot x^2 - 5 \cdot 2^x \cdot x = 14 \cdot 2^x$

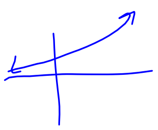
$2^x \cdot x^2 - 5 \cdot 2^x \cdot x - 14 \cdot 2^x = 0$

$2^x(x^2-5x-14) = 0$

$2^x(x-7)(x+2) = 0$

$2^x = 0$ $\quad x-7=0$ $\quad x+2=0$

never 0 $\quad \boxed{x=7}$ $\quad \boxed{x=-2}$



17. An account pays interest at a rate of 3% per year compounded continuously. If $3,000 is placed into this account today, how much will be in the account after 8 years?

$A = Pe^{rt}$

$r = .03$

$P = 3000$

$t = 8$

$A = ?$

$A = 3000e^{.03 \cdot 8}$

$\approx \boxed{\$3813.75}$

18. Susie puts her money into an account that pays 2.5% per year compounded continuously. If she has $5,400 after 13 years, how much did she originally put into the account?

$A = Pe^{rt}$

$r = .025$
$A = 5400$
$t = 13$
$P = ?$

$5400 = Pe^{.025(13)}$

$\boxed{P \approx \$3901.65}$

19. State the domain of each of the following functions using interval notation:

(a) $f(x) = \dfrac{2x^2+11x-21}{6x^3+45x^2+21x}$

$6x^3+45x^2+21x \neq 0$

$3x(2x^2+15x+7) \neq 0$

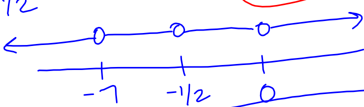
$3x(2x+1)(x+7) \neq 0$

$3x \neq 0 \quad 2x+1 \neq 0 \quad x+7 \neq 0$

$x \neq 0 \quad 2x \neq -1 \quad x \neq -7$

$x \neq -1/2$

Domain:
1. Can't divide by zero
2. The inside of an even root cannot be negative
3. The inside of a log must be positive.



$\boxed{(-\infty,-7)\cup(-7,-1/2)\cup(-1/2,0)\cup(0,\infty)}$

(b) $g(x) = \dfrac{\sqrt[8]{4x-3}}{e^x-3}$ even

$4x-3 \geq 0$

$4x \geq 3$

$x \geq 3/4$

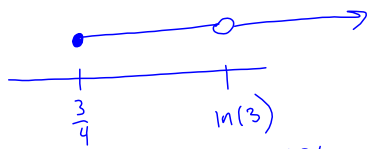
$e^x - 3 \neq 0$

$e^x \neq 3$

$\ln(e^x) \neq \ln(3)$

$x \cdot \underbrace{\ln(e)}_{1} \neq \ln(3)$

$x \neq \ln(3)$



≈ 1.0986

$\boxed{[3/4, \ln(3)) \cup (\ln(3), \infty)}$

(c) $h(x) = \dfrac{\ln(x)}{\sqrt[7]{x+3}}$ odd

$x > 0$

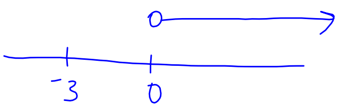
$x+3 \neq 0$

$x \neq -3$

$\sqrt[7]{x+3} \neq 0$

$\left((x+3)^{1/7}\right)^7 \neq (0)^7$

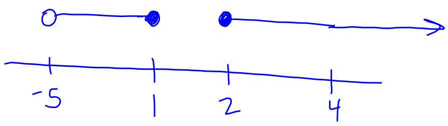
$x+3 \neq 0$



$\boxed{(0,\infty)}$

(d) $k(x) = \begin{cases} x^2+7-2^x & \text{if } -5 < x \leq 1 \\ \dfrac{x+2}{x^2-3x} & \text{if } 2 \leq x < 4 \\ \sqrt[4]{x-5} & \text{if } x \geq 4 \end{cases}$

The intervals on which $k(x)$ has a formula.



1st branch

x^2+7-2^x

No problems here

2nd branch

$\dfrac{x+2}{x^2-3x}$

$x^2-3x \neq 0$

$x(x-3) \neq 0$

$\cancel{x \neq 0} \quad x-3 \neq 0$

$x \neq 3$

not on the interval $2 \leq x < 4$

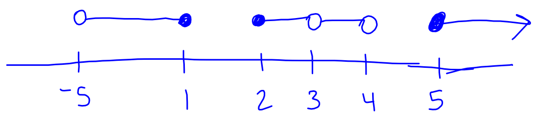
3rd branch

even $\sqrt[4]{x-5}$

$x-5 \geq 0$

$x \geq 5$

Domain of $k(x)$:

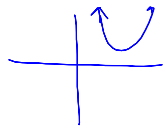


$\boxed{(-5,1] \cup [2,3) \cup (3,4) \cup [5,\infty)}$

20. Solve the following equations for the exact value(s) of x:

$ax^2+bx+c=0$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$



(a) $2x^2-7x+11=0$

$\cancel{(2x-11)(x-1)=0}$

can't factor into 2 linear terms here

$a=2$
$b=-7$
$c=11$

$x=\dfrac{+7\pm\sqrt{(-7)^2-4(2)(11)}}{2(2)}=\dfrac{7\pm\sqrt{-39}}{4}$ ← we can't take the square root of a negative #

$\boxed{\text{No solution}}$

(b) $\log_2(6x-1)+\log_2(x+2)=\log_2 19$

$\log_2\big((6x-1)(x+2)\big)=\log_2 19$

$(6x-1)(x+2)=19$ *

$6x^2+12x-x-2=19$

$6x^2+11x-2-19=0$

$6x^2+11x-21=0$

$(6x-7)(x+3)=0$

$6x-7=0 \quad x+3=0$

$6x=7 \quad x=-3$

$x=7/6$

OR

$\log_2\big((6x-1)(x+2)\big)-\log_2 19=0$

$\log_2\left(\dfrac{(6x-1)(x+2)}{19}\right)=0$

$2^0=\dfrac{(6x-1)(x+2)}{19}$

$1=\dfrac{(6x-1)(x+2)}{19}$

* $19=(6x-1)(x+2)$

Check $\boxed{x=7/6}$ ✓

$\log_2\big(\underbrace{\cancel{6}\cdot\tfrac{7}{\cancel{6}}-1}_{6>0}\big)+\log_2\big(\underbrace{\tfrac{7}{6}+2}_{19/6>0}\big)$

Check $\cancel{x=-3}$

$\log_2\big(\underbrace{6(-3)-1}_{-19}\big)+\log_2\big(\underbrace{-3+2}_{-1}\big)$

can't take the log of a negative #

(c) $3\cdot 4^x-5^x=0$

$\dfrac{3\cdot 4^x}{4^x}=\dfrac{5^x}{4^x}$

$3=\dfrac{5^x}{4^x}$

$3=\left(\dfrac{5}{4}\right)^x$

$\ln(3)=\ln\left(\left(\dfrac{5}{4}\right)^x\right)$

$\dfrac{\ln(3)}{\ln(5/4)}=\dfrac{x\cdot\cancel{\ln(5/4)}}{\cancel{\ln(5/4)}}$

$\boxed{x=\dfrac{\ln(3)}{\ln(5/4)}}$

(d) $\log_3 81-11^{\log_{11}5}+\log_4(4^x)=4^{3/2}$

$4-5+x=(4^{1/2})^3$

$-1+x=2^3$

$-1+x=8$

$\boxed{x=9}$

OR

$\sqrt{4^3}=(4^3)^{1/2}=64^{1/2}=8$

21. Simplify the following expressions.

Diff. quotient for $f(x) = \frac{2}{3x+5}$ $\left(\text{Recall: } \frac{f(x+h)-f(x)}{h}\right)$

(a) $\dfrac{\dfrac{2}{\underbrace{3(x+h)+5}_{3x+3h+5}} - \dfrac{2}{3x+5}}{h}$

$$= \frac{1}{h}\left(\frac{(3x+5)}{(3x+5)}\cdot\frac{2}{(3x+3h+5)} - \frac{2}{(3x+5)}\cdot\frac{(3x+3h+5)}{(3x+3h+5)}\right)$$

$$= \frac{1}{h}\left(\frac{2(3x+5)-2(3x+3h+5)}{(3x+5)(3x+3h+5)}\right)$$

$$= \frac{1}{h}\left(\frac{\cancel{6x}+\cancel{10}-\cancel{6x}-6h-\cancel{10}}{(3x+5)(3x+3h+5)}\right)$$

$$= \frac{1}{h}\left(\frac{-6h}{(3x+5)(3x+3h+5)}\right) = \frac{-6\cancel{h}}{\cancel{h}(3x+5)(3x+3h+5)} = \boxed{\frac{-6}{(3x+5)(3x+3h+5)}},\ h \neq 0$$

(b) $\dfrac{\sqrt[7]{x^3y^{-2}}\,(x^3yz^{-1})^{-2}}{(4x^2)^2yz}$

$$= \frac{(x^3y^{-2})^{1/7}\,(x^3)^{-2}\,y^{-2}\,(z^{-1})^{-2}}{4^2\,(x^2)^2\,y\,z} = \frac{(x^3)^{1/7}(y^{-2})^{1/7}\,x^{-6}\,y^{-2}\,z^2}{16\,x^4\,y\,z}$$

$$\frac{x^{3/7}\,y^{-2/7}\,x^{-6}\,y^{-2}\,z^2}{16x^4\,y\,z}$$

$$= \frac{1}{16}\,\frac{x^{3/7}x^{-6}}{x^4}\cdot\frac{y^{-2/7}y^{-2}}{y^1}\cdot\frac{z^2}{z^1}$$

$$= \frac{1}{16}\,x^{3/7-6-4}\,y^{-2/7-2-1}\,z^{2-1}$$

$$= \frac{1}{16}\,x^{-67/7}\,y^{-23/7}\,z$$

$$= \boxed{\frac{z}{16\,x^{67/7}\,y^{23/7}}}$$

22. The price-demand function for a particular product is $p(x) = 522 - 4x$ where $p(x)$ is the unit price when x units are demanded. The company making the product has a cost function of $C(x) = 42x + 13400$ where x is the number of items made and sold. Find the number of items the company must make and sell in order to maximize its profit.

$$R(x) = x(522 - 4x)$$
$$= 522x - 4x^2$$
$$= -4x^2 + 522x$$

$$P(x) = R(x) - C(x)$$
$$= -4x^2 + 522x - (42x + 13400)$$
$$= -4x^2 + 522x - 42x - 13400$$
$$= -4x^2 + 480x - 13400$$

$a = -4$
$b = 480$
$c = -13400$

$$h = \frac{-b}{2a} = \frac{-480}{2(-4)} = 60$$

$$k = P(60) = -4(60)^2 + 480(60) - 13400$$
$$= 1000$$

$\boxed{\text{60 items would yield a maximum profit of \$1000}}$

23\. A taxpayer in a particular country is taxed as follows. The income between \$0 and \$8,025, inclusive, is taxed at 10%. Any income over \$8,025 is taxed at 15%. Find a piecewise-defined function, $T(x)$, that will allow a taxpayer to determine the total amount of tax owed if they have x dollars of income.

$$T(x) = \begin{cases} .1x, & 0 \le x \le 8025 \\ .1(8025) + .15(x - 8025), & x > 8025 \end{cases}$$

$= 802.5 + .15x - 1203.75$

$= .15x - 401.25$

If we had an income of \$8055,

$.1(8025) + .15(30)$

$8055 - 8025$

24. An account pays interest at the rate of 5% per year compounded continuously. How long will it take for the amount placed into the account to double?

$A = Pe^{rt}$ $\quad r = .05$ $\quad P = 100$

$\frac{2P}{P} = \frac{Pe^{.05t}}{P}$ $\quad t = ?$ $\quad A = 200$

$A = 2P$ $\quad$ OR $\quad \frac{200}{100} = \frac{100e^{.05t}}{100}$

$2 = e^{.05t}$ $\quad 2 = e^{.05t}$

$\ln(2) = \ln(e^{.05t})$

$\ln(2) = .05t \cdot \underbrace{\ln(e)}_{1}$

$\frac{\ln(2)}{.05} = \frac{.05t}{.05}$

$\boxed{13.8629 \text{ years}} \approx \frac{\ln(2)}{.05} = t$

25. If $g(x) = -3f(x-2)+4$, describe how the graph of $f(x)$ would need to be transformed to get $g(x)$.

- shift the graph of $f(x)$ 2 units to the right
- reflect the graph about the x-axis
- stretch the graph vertically by a factor of 3
- shift the graph 4 units up.

26. If the graph of $f(x) = x^2$ is shifted right 3 units, vertically stretched by a factor of 4, reflected about the x-axis and then shifted 2 units down, what is the equation of the resulting graph?

$$\boxed{g(x) = -4(x-3)^2 - 2}$$

27. Use the table below to evaluate the following:

x	1	2	4	7	9	12
$f(x)$	-3	12	7	-2	10	5
$g(x)$	9	-4	11	1	5	-8

(a) $f(g(1)) = f(9) = \boxed{10}$

(b) $g(f(4)) = g(7) = \boxed{1}$

(c) $g(f(12)-1) = g(5-1) = g(4) = \boxed{11}$

28. Expand the following function using properties of logarithms:

$$f(x) = \log_3\left(\frac{(2x-7)^3(4x-3)}{(x+5)\sqrt{8x-9}}\right)$$

$$= \log_3\left((2x-7)^3(4x-3)\right) - \log_3\left((x+5)\sqrt{8x-9}\right)$$

$$= \log_3\left((2x-7)^3\right) + \log_3(4x-3) - \left(\log_3(x+5) + \log_3\left((8x-9)^{1/2}\right)\right)$$

$$= \boxed{3\log_3(2x-7) + \log_3(4x-3) - \log_3(x+5) - \frac{1}{2}\log_3(8x-9)}$$

29. Find the zeros of $f(x) = \dfrac{(x-2)(x+3)}{(2x-7)(5x-2)}$.

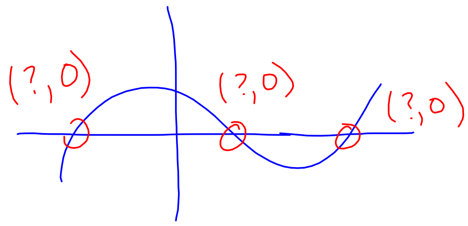
looking for the values of x that make the numerator 0 but not the denominator.

$f(x) = 0$

$(x-2)(x+3) = 0$

$x-2=0 \qquad x+3=0$

$\boxed{x=2} \qquad \boxed{x=-3}$



Multiple Choice and True/False Questions:

30. Determine whether each of the following statments is True or False.

(a) For a quadratic function $f(x) = ax^2 + 5x + 2$, the minimum value of the function occurs at $x = \frac{-5}{2a}$ for any value o f a. False

(b) The graph of a polynomial is continuous for all real numbers. True

(c) If a polynomial of even degree has a negative leading coefficient, the function value will approach negative infinity as x approaches negative infinity. True

(d) If a polynomial of odd degree has a positive leading coefficient, the function value will approach positive infinity as x approaches positive infinity. True

(e) The domain of $\sqrt[7]{7x-3}$ is $[\frac{3}{7}, \infty)$ False

(f) $\sqrt[12]{x^{-2}y^3} = x^{-1/6}y^{1/4}$ True

(g) The function, $f(x) = 3^x$, has a zero at $x = 0$. False

(h) $4^{\ln 4} = e$. False

(i) If $f(1) = 4$, $f(4) = 3$, $g(2) = -2$, and $g(3) = 7$ then $g(f(4)) = 1$. False

(j) If $f(x) = 2x^2 + 4x - 7$ then $f(x+h) = 2x^2 + 4x - 7 + h$. False

31. The cost, in dollars, of manufacturing x units of a product is given by $C(x) = 4x + 15000$. The demand equation for the same product is given by $p = -\frac{1}{500}x + 22$ where x is the quantity demanded at a unit price of $\$p$. If the selling price of the item is determined by the demand function, what is the maximum profit that this manufacturer can obtain?

(a) \$15,000

(b) \$25,000

(c) \$30,000

(d) \$25,500

(e) None of the above

32. If $f(x) = 3x^2 - 4x + 7$, find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

(a) $6x + 3h - 4$
(b) $3h - 4$
(c) $\dfrac{-2x^2 + 2xh + h^2 - 4h}{h}$
(d) 1
(e) None of the above

33. The graph of a quadratic function that has a vertex of $(3, -3)$, opens down, and passes through the point $(2, -5)$ has a y-intercept of

(a) -3
(b) -12
(c) -21
(d) 15
(e) None of the above

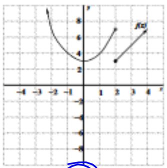
34. What is the domain of $\dfrac{\sqrt{x+100}}{5 + \sqrt[3]{2x-7}}$?

(a) $[-100, -59) \cup (-59, \infty)$
(b) $[3.5, \infty)$
(c) $[-100, 3.5) \cup (3.5, \infty)$
(d) $[-100, \infty)$
(e) None of the above

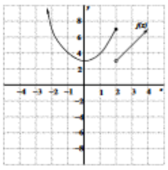
35. Which of the following is equivalent to $\dfrac{3^x 4^{x-2}}{2^x 9^{3x}}$?

(a) $\dfrac{3^{2x}}{4}$
(b) $\dfrac{1}{16}\left(\dfrac{3}{2}\right)^x\left(\dfrac{4}{9}\right)^{3x}$
(c) $\dfrac{12^{2x-2}}{18^{4x}}$
(d) $\dfrac{2^{x-4}}{3^{5x}}$
(e) None of the above

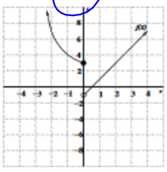
36. If $f(x) = \begin{cases} x^2+3 & \text{if } x < 2 \\ 2x-1 & \text{if } x \geq 2 \end{cases}$, which graph below represents $f(x)$?



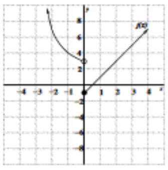
(a)



(b)



(c)



(d)

37. Determine the domain of the following function.

$$f(x) = \frac{\sqrt[3]{x-3}}{x^2-7x+12}$$

(a) $(3,4)\cup(4,\infty)$
(b) $[3,4)\cup(4,\infty)$
(c) $(-\infty,4)\cup(4,\infty)$
(d) $(-\infty,3)\cup(3,4)\cup(4,\infty)$
(e) None of the above

38. What amount will an account have after 10 years if \$3,000 is invested at an annual rate of 3.15% compounded continuously? (Round to the nearest penny.)

(a) \$7,110.78
(b) \$4,109.08
(c) \$70,008.19
(d) \$4,110.78
(e) None of the above

39. The demand for a particular flashlight is given by $p = -\frac{3}{1000}x + 12$ where x is the number of flashlights demanded at a unit price of p dollars. If the demand function determines the price that the manufacturer sells the flashlight at, what unit price will maximize the revenue?

(a) \$6
(b) \$8
(c) \$2,000
(d) \$12,000
(e) None of the above

40. Given $g(x) = -x^2 + 3x - 4$, find $\frac{g(x+h) - g(x)}{h}$

(a) $\frac{2x^3 + 2xh + h^2 + 3h}{h}$

(b) 1

(c) $3 - h + 2x$

(d) $-2x - h + 3$

(e) None of the above

41. What is the domain of $f(x) = \frac{\sqrt{3x-7}}{\sqrt[9]{13-4x}}$?

(a) $\left[\frac{7}{3}, \frac{13}{4}\right) \cup \left(\frac{13}{4}, \infty\right)$

(b) $\left[\frac{7}{3}, \infty\right)$

(c) $\left[\frac{13}{4}, \infty\right)$

(d) $\left(\frac{7}{3}, \frac{13}{4}\right) \cup \left(\frac{13}{4}, \infty\right)$

(e) None of the above

42. If $f(x) = 2x^3 - 7x + 5$ and $g(x) = \sqrt[5]{9x^3}$. Which of the following is equivalent to $f(g(x))$?

(a) $2 \cdot 9^{3/5} x^{9/5} - 7 \cdot 9^{1/5} x^{3/5} + 5$

(b) $9^{1/5}(2x^3 - 7x + 5)^{3/5}$

(c) $2\sqrt[5]{9x} - 7\sqrt[5]{9x^3} + 5$

(d) $9(2x^3 - 7x + 5)^{3/5}$

(e) None of the above

43. Solve the following equation for x:
$\log_8(2x + 11) + \log_8(x) = \log_8(6)$

(a) $x = 0.5$ and $x = -6$ only

(b) $x = -6$ only

(c) $x = 0.5$ only

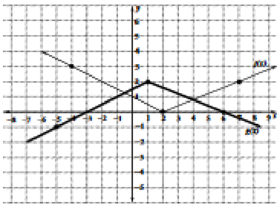
(d) $x = -11/2$ and $x = 6$ only

(e) None of the above

44. If $\log_b 2 = 0.2789$ and $\log_b 3 = 0.4421$, what is $\log_b \left(\frac{b^4}{18}\right)$?

(a) 1.1631
(b) 2.8369
(c) 4.6053
(d) 3.3947
(e) None of the above

45. Given the graphs of $f(x)$ and $g(x)$ below, write $g(x)$ in terms of $f(x)$.



(a) $g(x) = -f(x+1)+2$
(b) $g(x) = -f(x+2)+1$
(c) $g(x) = -f(x-1)+2$
(d) $g(x) = f(x+1)+2$
(e) None of the above

46. The expression $\frac{2e^{x+4}}{4^{x+2}e^{2x-7}}$ is equivalent to which of the following?

(a) $2^{-2x-3}e^{-x+11}$
(b) $4^{-x}e^{-x+11}$
(c) $2^{-2x-4}e^{-x+11}$
(d) $2^{-2x+5}e^{-x-3}$
(e) $2^{5-2x}e^{-x+11}$